

## On the $\eta'$ Gluonic Admixture

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### Abstract

The  $\eta'$  which is an  $SU(3)_F$  singlet state can contain a pure gluon component, gluonium. We examine this possibility by analysing all available experimental data. It is pointed out that the  $\eta'$  gluonic component may be as large as 26%. We also show that the amplitude for  $J/\psi \rightarrow \eta'\gamma$  decay obtains a notable contribution from gluonium.

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# 1 Introduction

The CLEO collaboration reported an unexpectedly large branching ratio for  $B \rightarrow \eta' X_s$  [1]. One of the suggested mechanisms [2–13] to explain this problem considers the process  $b \rightarrow sg$ ,  $g \rightarrow \eta' g$  [2–7]. This mechanism is based on the anomalous coupling of  $gg \rightarrow \eta'$  which accounts for the large branching ratio for  $J/\psi \rightarrow \eta' \gamma$  decay. It should be noted that the gluonic component of  $\eta'$  has been studied extensively in the literature [14–21]. We shall determine the gluonic component of  $\eta'$  considering all known experimental data.

It is believed that  $\eta'$  consists of the  $SU(3)_F$  singlet and octet  $q\bar{q}$  states which we denote as  $\eta_1$  and  $\eta_8$ , respectively, and dominated by the singlet state. The  $SU(3)_F$  singlet state, differing from the octet state, can be composed of pure gluon states. Therefore, we examine another singlet state in  $\eta'$  made only of gluons, which we call gluonium.

The remainder of the paper is organized as follows. In Section 2, we describe our notation and introduce the gluonic component. The formalism for studying the radiative light meson decays is presented in Section 3. The recent discussions on the definition of the decay constants for  $\eta$  and  $\eta'$  [22–24] are taken into account. We then proceed to obtain the pseudoscalar mixing angle  $\theta_p$  and the possible gluonic content of  $\eta'$  in Section 4. The investigation of the radiative  $J/\psi$  decay is performed in Section 5. A summary and conclusions are given in Section 6.

## 2 Notation

$SU(3)_F \times U(1)$  symmetry introduces the pseudoscalar octet state  $\eta_8$  and singlet state  $\eta_1$  as

$$\begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix} = \begin{pmatrix} \sin \theta_I & -\cos \theta_I \\ \cos \theta_I & \sin \theta_I \end{pmatrix} \begin{pmatrix} \frac{u\bar{u}+d\bar{d}}{\sqrt{2}} \\ s\bar{s} \end{pmatrix} \quad (1)$$

where  $\theta_I$  is the ideal mixing angle which satisfies  $\theta_I = \tan^{-1} \frac{1}{\sqrt{2}}$ . The two physical states  $\eta$  and  $\eta'$  are considered as mixtures of these states with pseudoscalar mixing angle  $\theta_p$

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos \theta_p & -\sin \theta_p \\ \sin \theta_p & \cos \theta_p \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix} \quad (2)$$

Combining Eqs. (1) and (2), we rewrite

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos \alpha_p & -\sin \alpha_p \\ \sin \alpha_p & \cos \alpha_p \end{pmatrix} \begin{pmatrix} \frac{u\bar{u}+d\bar{d}}{\sqrt{2}} \\ s\bar{s} \end{pmatrix} \quad (3)$$

with  $\alpha_p = \theta_p - \theta_I + \frac{\pi}{2}$  which represents the discrepancy of the mixing angle from the ideal one. Note that the  $\phi$  and  $\omega$  in the vector meson system mix almost ideally, that is,  $\alpha_v \simeq 0$ . This characteristic deviation from the ideal mixing in  $\eta - \eta'$  system can be understood in terms of the anomaly. Let us take the derivative of the singlet axial vector current

$$\partial_\mu j^{\mu 5} = 2imq\gamma_5 \bar{q} - \frac{3\alpha_s}{4\pi} G_{\alpha\beta} \tilde{G}^{\alpha\beta} \quad (4)$$

where  $G_{\alpha\beta}$  is a gluonic field strength and  $\tilde{G}^{\alpha\beta}$  is its dual. The term proportional to  $G\tilde{G}$  is coming from the triangle anomaly [25]. It affects neither the octet axial vector nor the vector current. Eq. (4) implies that the pseudoscalar singlet state can be composed not only of  $q\bar{q}$  but also of gluons. Treating the gluon composite equivalent to the quark composite, the  $\eta'$  which is mostly  $SU(3)_F$  singlet may contain the pure gluon state, gluonium. Therefore, we reconstruct  $\eta - \eta'$  system by including gluonium. Then Eq. (2) is extended to a  $3 \times 3$  matrix with 3 mixing angles

$$\begin{pmatrix} \eta \\ \eta' \\ i \end{pmatrix} = \begin{pmatrix} \cos \theta_p \cos \gamma + \sin \theta_p \cos \phi \sin \gamma & -\sin \theta_p \cos \gamma + \cos \theta_p \cos \phi \sin \gamma & \sin \phi \sin \gamma \\ \cos \theta_p \sin \gamma + \sin \theta_p \cos \phi & \sin \theta_p \sin \gamma + \cos \theta_p \cos \phi \cos \gamma & \sin \phi \cos \gamma \\ -\sin \theta_p \sin \phi & -\cos \theta_p \sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_1 \\ gluonium \end{pmatrix}$$

where  $i$  is a "glueball-like state" which we refrain from discussing here. Since the mass of  $\eta$  is about the mass of  $\eta_8$  which is obtained from Gell-Mann Okubo mass formula, we assume that  $\eta$  does not contain the extra singlet state gluonium. Setting  $\gamma = 0$ , we obtain

$$\begin{pmatrix} \eta \\ \eta' \\ i \end{pmatrix} = \begin{pmatrix} \cos \theta_p & -\sin \theta_p & 0 \\ \sin \theta_p \cos \phi & \cos \theta_p \cos \phi & \sin \phi \\ -\sin \theta_p \sin \phi & -\cos \theta_p \sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_1 \\ gluonium \end{pmatrix}. \quad (5)$$

It is convenient to write the  $\eta$  and  $\eta'$  states as [14]

$$|\eta\rangle = X_\eta \left| \frac{u\bar{u} + d\bar{d}}{\sqrt{2}} \right\rangle + Y_\eta |s\bar{s}\rangle \quad (6)$$

$$|\eta'\rangle = X_{\eta'} \left| \frac{u\bar{u} + d\bar{d}}{\sqrt{2}} \right\rangle + Y_{\eta'} |s\bar{s}\rangle + Z_{\eta'} |gluonium\rangle. \quad (7)$$

$X_{\eta(\eta')}$ ,  $Y_{\eta(\eta')}$  and  $Z_{\eta'}$  are normalized as

$$X_\eta^2 + Y_\eta^2 = 1 \quad (8)$$

$$X_{\eta'}^2 + Y_{\eta'}^2 + Z_{\eta'}^2 = 1 \quad (9)$$

and relate to the mixing angles

$$X_\eta = \cos \alpha_p, \quad Y_\eta = -\sin \alpha_p, \quad (10)$$

$$X_{\eta'} = \cos \phi \sin \alpha_p, \quad Y_{\eta'} = \cos \phi \cos \alpha_p, \quad Z_{\eta'} = \sin \phi. \quad (11)$$

### 3 Decay rates

We calculate the decay rates by using the vector meson dominance model (VDM) and the  $SU(3)_F$  quark model (see for example, [26–28]). In this method, the decay rates are expressed in terms of the masses and the decay constants of light mesons. The decay constants for vector mesons which are defined by

$$m_V f_V \epsilon^\mu = \langle 0 | j_V^\mu | V(p, \lambda) \rangle \quad (12)$$

are well determined by their decays into  $e^+e^-$  [29] as

$$f_\rho = (216 \pm 5) \text{MeV}, \quad f_\omega = (195 \pm 3) \text{MeV}, \quad f_\phi = (237 \pm 4) \text{MeV}. \quad (13)$$

On the other hand, the decay constants for  $\eta$  and  $\eta'$  are not well-defined because of the anomaly. Recently, there has been considerable progress on the parametrization of the decay constants of  $\eta - \eta'$  system [22–24]. Following Reference [24], we utilize the decay constants defined by

$$if_x p_\mu = \langle 0 | u \gamma^\mu \gamma_5 \bar{u} + d \gamma^\mu \gamma_5 \bar{d} | \frac{u\bar{u} + d\bar{d}}{\sqrt{2}} \rangle \quad (14)$$

$$if_y p_\mu = \langle 0 | s \gamma^\mu \gamma_5 \bar{s} | s\bar{s} \rangle \quad (15)$$

which are considered as the decay constants for the  $SU(3)_F$  singlet states at non-anomaly limit. Since the state  $|\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}\rangle$  in Eq. (14) is equivalent to  $\pi^0$  but an isospin singlet, we can approximately have the following relation by assuming that the isospin breaking effect is not large:

$$f_x = f_\pi.$$

When  $SU(3)_F$  symmetry is exact  $f_y$  in Eq. (15) is equal to  $f_x$ . However, the mass difference between the  $u$  and  $d$  quarks and the  $s$  quark is notable. The Gell-Mann-Okubo mass formula gives a quantitative estimate of the  $s$  quark mass breaking effect. Similarly, this breaking effect for our decay constants can be included through

$$f_y = \sqrt{2f_K^2 - f_\pi^2}.$$

The known values for  $f_\pi = 131$  MeV and  $f_K = 160$  MeV lead to

$$f_x = 131 \text{ MeV}, \quad f_y = 1.41 \times 131 \text{ MeV}. \quad (16)$$

It is shown in Reference [24] that the approximate values in Eq. (16) are justified phenomenologically and also satisfy the result of chiral perturbation theory in [22].

Using these decay constants, the radiative decay rates of the light mesons can be written in terms of  $X_{\eta(\eta')}$ ,  $Y_{\eta(\eta')}$  and  $Z_{\eta'}$  in the VDM as follows,

$$\Gamma(\omega \rightarrow \eta\gamma) = \frac{\alpha}{24} \left( \frac{m_\omega^2 - m_\eta^2}{m_\omega} \right)^3 \left( \frac{m_\omega}{f_\omega \pi^2} \right)^2 \left( \frac{X_\eta}{4f_x} \right)^2 \quad (17)$$

$$\Gamma(\phi \rightarrow \eta\gamma) = \frac{\alpha}{24} \left( \frac{m_\phi^2 - m_\eta^2}{m_\phi} \right)^3 \left( \frac{m_\phi}{f_\phi \pi^2} \right)^2 \left( -2 \frac{Y_\eta}{4f_y} \right)^2 \quad (18)$$

$$\Gamma(\eta \rightarrow \gamma\gamma) = \frac{\alpha^2}{288\pi^3} m_\eta^3 \left( \frac{5X_\eta}{f_x} + \frac{\sqrt{2}Y_\eta}{f_y} \right)^2 \quad (19)$$

$$\Gamma(\eta' \rightarrow \omega\gamma) = \frac{\alpha}{8} \left( \frac{m_{\eta'}^2 - m_\omega^2}{m_{\eta'}} \right)^3 \left( \frac{m_\omega}{f_\omega \pi^2} \right)^2 \left( \frac{X_{\eta'}}{4f_x} \right)^2 \quad (20)$$

$$\Gamma(\eta' \rightarrow \rho\gamma) = \frac{\alpha}{8} \left( \frac{m_{\eta'}^2 - m_\rho^2}{m_{\eta'}} \right)^3 \left( \frac{m_\rho}{f_\rho \pi^2} \right)^2 \left( \frac{3X_{\eta'}}{4f_x} \right)^2 \quad (21)$$

$$\Gamma(\phi \rightarrow \eta'\gamma) = \frac{\alpha}{24} \left( \frac{m_\phi^2 - m_{\eta'}^2}{m_\phi} \right)^3 \left( \frac{m_\phi}{f_\phi \pi^2} \right)^2 \left( -2 \frac{Y_{\eta'}}{4f_y} \right)^2 \quad (22)$$

$$\Gamma(\eta' \rightarrow \gamma\gamma) = \frac{\alpha^2}{288\pi^3} m_{\eta'}^3 \left( \frac{5X_{\eta'}}{f_x} + \frac{\sqrt{2}Y_{\eta'}}{f_y} \right)^2 \quad (23)$$

where the OZI suppressed process occurring from  $\phi - \omega$  mixing violation is ignored. In fact this breaking effect is expected to be very small; for example, in the case of the  $\phi \rightarrow \pi^0\gamma$  decay,  $\sin\alpha_V$  is estimated to be less than 0.02.

It is known that the VDM works quite well in the describing decay modes (see, for example, Refs. [30–32]). This is supported by performing the computation of the decay rates  $\omega \rightarrow \pi^0\gamma$  and  $\pi^0 \rightarrow \gamma\gamma$  which do not depend on  $X_{\eta(\eta')}$ ,  $Y_{\eta(\eta')}$  and  $Z_{\eta'}$ :

$$\Gamma(\omega \rightarrow \pi^0\gamma) = \frac{\alpha}{24} \left( \frac{m_\omega^2 - m_{\pi^0}^2}{m_\omega} \right)^3 \left( \frac{m_\omega}{f_\omega \pi^2} \right)^2 \left( \frac{3}{4f_{\pi^0}} \right)^2 = 0.72 \text{ MeV} \quad (24)$$

$$\Gamma(\pi^0 \rightarrow \gamma\gamma) = \frac{\alpha^2}{288\pi^3} m_{\pi^0}^3 \left( \frac{3}{f_{\pi^0}} \right)^2 = 0.0077 \text{ KeV}. \quad (25)$$

which are rather consistent with the experimental data [29]

$$\Gamma(\omega \rightarrow \pi^0\gamma) = (0.72 \pm 0.043) \text{ MeV},$$

$$\Gamma(\pi^0 \rightarrow \gamma\gamma) = (0.0077 \pm 0.00055) \text{ KeV},$$

respectively. Here we used  $f_{\pi^0} = 131 \text{ MeV}$ . In the case of the  $\rho^0 \rightarrow \pi^0\gamma$  decay, the model calculation gives  $\Gamma(\rho^0 \rightarrow \pi^0\gamma) = 0.06 \text{ MeV}$  which is small compared to the experimental value  $\Gamma(\rho^0 \rightarrow \pi^0\gamma) = (0.10 \pm 0.026) \text{ MeV}$ . We note, however, that  $\rho^0 \rightarrow \pi^0\gamma$  decay rate still has a large error. It would be discussed in detail as more data will be available. We expect that the theoretical uncertainty occurring from the VDM is less than 15%. This number is within the range of the error estimated in [33] according to a QCD-based method.

## 4 Results

### 4.1 Results for $X_\eta$ and $Y_\eta$ (determination of $\theta_p$ )

First, we analyse  $\omega \rightarrow \eta\gamma$ ,  $\eta \rightarrow \gamma\gamma$  and  $\phi \rightarrow \eta\gamma$  decays. Substituting the left hand side of Eq.(17)  $\sim$  (19) for the experimental data and the errors [29], we obtain the constraint on  $X_\eta$  and  $Y_\eta$  and consequently,  $\alpha_p$  via Eq. (10). The result is shown in Figure 1. The circumference denotes the constraint for  $X_\eta$  and  $Y_\eta$  in Eq. (8). As we estimated in the previous section, the theoretical error of 15% is included.

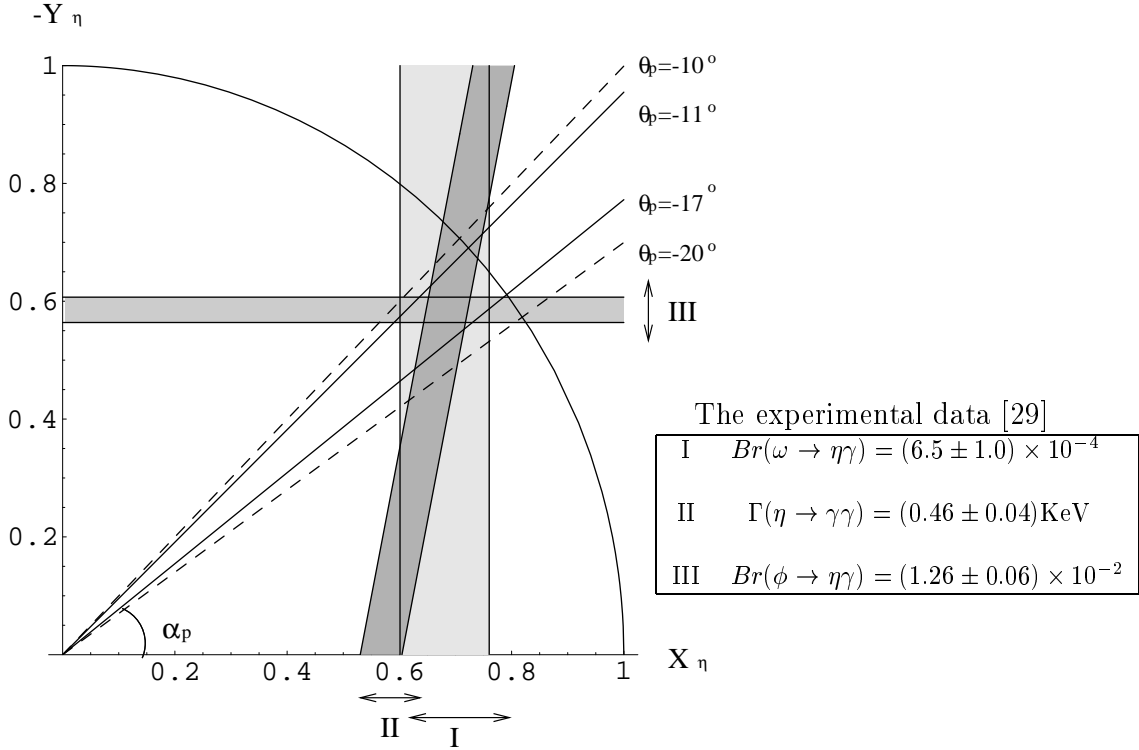


Figure 1: The experimental bounds for  $\omega \rightarrow \eta\gamma$  (I),  $\eta \rightarrow \gamma\gamma$  (II) and  $\phi \rightarrow \eta\gamma$  (III). The condition for  $X_\eta$  and  $Y_\eta$  in Eq. (8) is shown as a circumference. We obtain a constraint  $-17^\circ < \theta_p < -11^\circ$ .

In Figure 1, we have plotted simply the averages in the Review of Particle Physics [29]. However, the experiments still have large errors for these processes. Looking carefully at the data in [29], we analyse the result depicted in Figure 1. A result for  $\eta \rightarrow \gamma\gamma$  decay in 1974,  $\Gamma(\eta \rightarrow \gamma\gamma) = (0.32 \pm 0.046)\text{KeV}$ , is inconsistent with all other experiments so that we excluded this result when averaging. Consequently, the central value of  $\Gamma(\eta \rightarrow \gamma\gamma)$  gets an increase of 5%, which leads the bound II in Figure 1 to shift to the right by about 0.03. After the shift, the bound II intersects the circle between  $\alpha_p \simeq -44^\circ$  and  $-41^\circ$  and we obtain the result from the  $\eta \rightarrow \gamma\gamma$  decay as  $\theta_p \simeq -14^\circ \sim -11^\circ$ . Similarly, a result for  $\omega \rightarrow \eta\gamma$  in 1977, which is  $Br(\omega \rightarrow \eta\gamma) = (3.0^{+2.5}_{-1.8}) \times 10^{-4}$ , is small compared to other data and in fact, it has a 70% error. Exclusion of this value leads to a 6% increase of the center value and about a 0.04 shift to the right of the bound I in Figure 1. As a result, the bound I intersects the circle at  $\theta_p \simeq -17^\circ \sim -8^\circ$ . Finally, the experiment in 1983 of  $\phi \rightarrow \eta\gamma$  reports a branching ratio  $Br(\phi \rightarrow \eta\gamma) = (0.88 \pm 0.20) \times 10^{-2}$  which is smaller than any other vlues. We exclude this result and obtain a 0.01 upward shift of the bound III in Figure 1. Then the result for  $\theta_p$  from  $\phi \rightarrow \eta\gamma$  is  $-20^\circ \sim -11^\circ$ .

Eventually, we conclude that the experimental result for  $\theta_p$  converges in a range of  $-17^\circ \sim -11^\circ$ . Note that we obtained a smaller value of  $|\theta_p|$  than the previous work [14] which gave  $-21^\circ < \theta_p < -16^\circ$ . The change is mainly caused by two facts: the average of the decay rate of  $\eta \rightarrow \gamma\gamma$  became smaller, and we utilized differently defined decay constants for  $\eta$  and  $\eta'$ .

## 4.2 Result for $X_{\eta'}$ , $Y_{\eta'}$ and $Z_{\eta'}$ (determination of $Z_{\eta'}$ )

Now we analyse  $\eta' \rightarrow \omega\gamma$ ,  $\eta' \rightarrow \rho\gamma$ ,  $\eta' \rightarrow \gamma\gamma$  and  $\phi \rightarrow \eta'\gamma$  decays. Constraints on  $X_{\eta'}$ ,  $Y_{\eta'}$  and  $Z_{\eta'}$  can be obtained by using Eqs. (20)  $\sim$  (23). The experimental bounds [29] for these decays are shown in Figure 2. As in the case of  $\eta$ , a 15 % theoretical error is taken into account. From the analysis in Section 4.1, we have a constraint on  $\theta_p$  between  $-17^\circ$  and  $-11^\circ$ . Since we have a relation  $X_{\eta'}^2 + Y_{\eta'}^2 + Z_{\eta'}^2 = 1$ , the result  $X_{\eta'}^2 + Y_{\eta'}^2 < 1$  represents  $\eta'$  having a gluonic component.

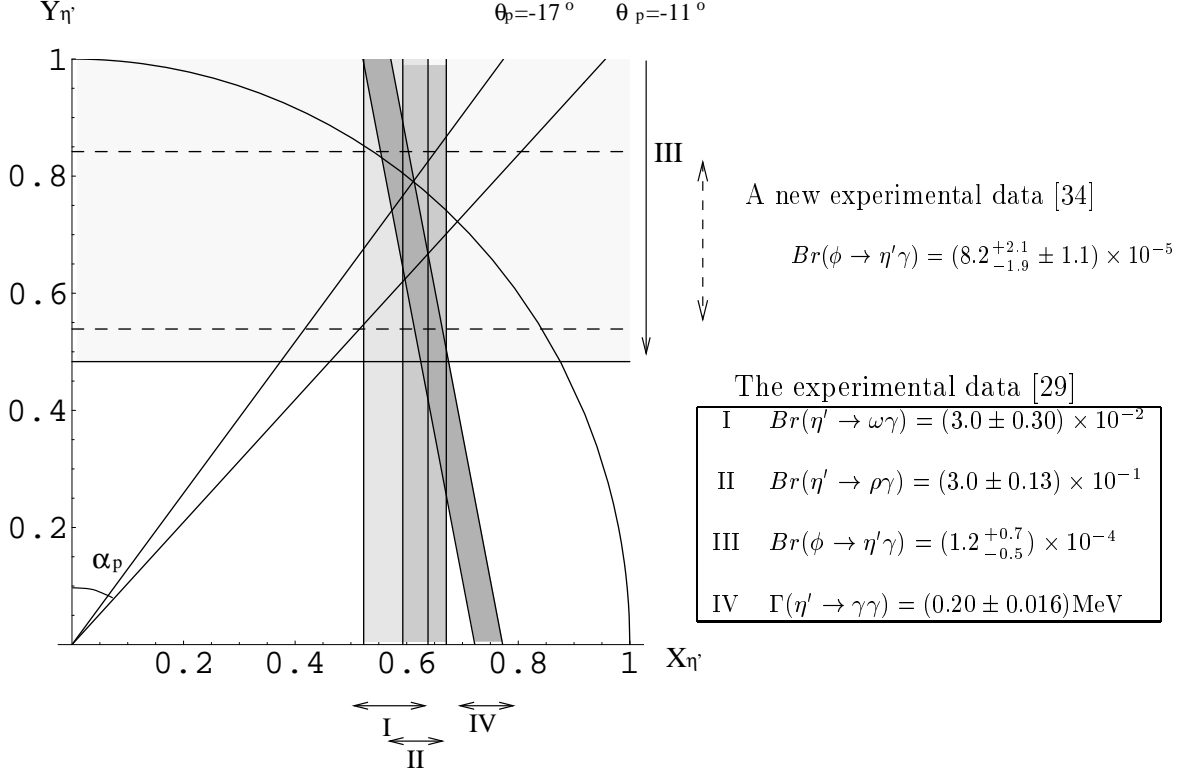


Figure 2: The experimental bounds for  $\eta' \rightarrow \omega \gamma$  (I),  $\eta' \rightarrow \rho \gamma$  (II),  $\phi \rightarrow \eta' \gamma$  (III) and  $\eta' \rightarrow \gamma \gamma$  (IV). The dashed line shows the bound for  $\phi \rightarrow \eta' \gamma$  from a new experimental data [34]. Taking the value of  $\theta_p$  to be  $-11^\circ$  which is allowed in Figure 1, we observe the maximum 26% of the gluonic component in  $\eta'$ .

We have the following observations:

- ★ The maximum gluonic admixture in  $\eta'$  is obtained to be 6% for  $\theta_p = -17^\circ$ , 17% for  $\theta_p = -14^\circ$  and 26% for  $\theta_p = -11^\circ$  where the percentage is computed by

$$R = \frac{Z_{\eta'}}{X_{\eta'} + Y_{\eta'} + Z_{\eta'}}. \quad (26)$$

- ★ If future experiments show an increase of 10% in the central values of the  $\eta' \rightarrow \rho \gamma$  or  $\eta' \rightarrow \gamma \gamma$  decay rate, the existence of the gluonic content in  $\eta'$  will be excluded for large  $|\theta_p|$ .
- ★ The CMD-2 collaboration observed  $\phi \rightarrow \eta' \gamma$  in 1999. Using their new result [34]

$$Br(\phi \rightarrow \eta' \gamma) = (8.2^{+2.1}_{-1.9} \pm 1.1) \times 10^{-5},$$

the dashed bound in Figure 2 is obtained. The new data show that the observation of the maximum gluonic admixture described above is still allowed. A more stringent constraint is expected once the data from the  $\phi$  factory at DAΦNE come out.



## 5 $J/\psi$ decays

Now we analyse the radiative  $J/\psi$  decays into  $\eta$  and  $\eta'$  and see the influence of the allowed amount of gluonic admixture in Section 4.2 on the amplitudes. The ratio of the two decay rates  $R_{J/\psi}$  can be written as [24, 35–37]

$$R_{J/\psi} = \frac{\Gamma(J/\psi \rightarrow \eta\gamma)}{\Gamma(J/\psi \rightarrow \eta'\gamma)} = \left( \frac{1 - m_\eta^2/m_{J/\psi}^2}{1 - m_{\eta'}^2/m_{J/\psi}^2} \right)^3 \left| \frac{\sqrt{2}\xi X_\eta - \zeta(-Y_\eta)}{(\sqrt{2}\xi X_{\eta'} + \zeta Y_{\eta'}) + g'_r Z_{\eta'}} \right|^2 \quad (27)$$

where  $\xi$ ,  $\zeta$  and  $g'_r$  are  $f_\pi/f_x$ ,  $f_\pi/f_y$ , and the coupling of two gluons to gluonium, respectively. Using the average of [29], we have

$$R_{J/\psi} = \frac{\Gamma(J/\psi \rightarrow \eta\gamma)}{\Gamma(J/\psi \rightarrow \eta'\gamma)} = 0.20 \pm 0.02. \quad (28)$$

The terms  $\sqrt{2}\xi X_{\eta^{(\prime)}}$  and  $\zeta Y_{\eta^{(\prime)}}$  in Eq.(27) represent the contributions from such intermediate processes as  $gg \rightarrow (u\bar{u}, d\bar{d} \text{ triangle loop}) \rightarrow \eta^{(\prime)}$  and  $gg \rightarrow (s\bar{s} \text{ triangle loop}) \rightarrow \eta^{(\prime)}$ , respectively (see Figure 3(a)) and the term  $g'_r Z_{\eta'}$  from  $gg \rightarrow (\text{gluonium}) \rightarrow \eta'$  (see Figure 3(b)). We define the ratio between the amplitudes for the process Figure 3(b) and Figure 3(a) by  $r$ :

$$r = \frac{g'_r Z_{\eta'}}{(\sqrt{2}\xi X_{\eta'} + \zeta Y_{\eta'})} \quad (29)$$

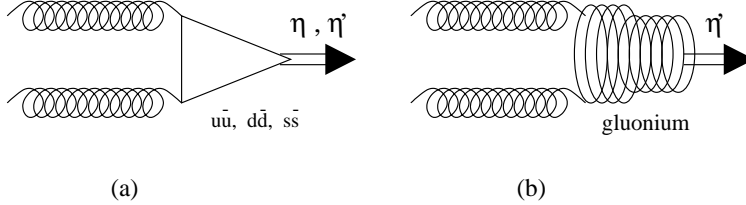


Figure 3: Coupling of  $\eta$  and  $\eta'$  to two gluons through quark and anti-quark triangle loop (a) and through gluonic admixture (b).

First, we examine the case of  $r = 0$  which means that gluonium does not contribute to  $J/\psi \rightarrow \eta'\gamma$  amplitude. In this case, the right hand side of Eq. (27) depends on only one parameter  $\alpha_p$ , so using Eq. (28),  $\theta_p$  can be determined. The result is shown in Figure 4. We observe that for  $g'_r Z_{\eta'} = 0$ , the  $\theta_p$  angle is determined in a region  $\theta_p = -13^\circ \pm 1.0^\circ$ . On the other hand, in the analysis of the glue content in Section 4.2,  $Z_{\eta'} = 0$  is allowed only when  $\theta_p$  is in a narrow region around  $-17^\circ$  (see Figure 2). This disagreement indicates that  $Z_{\eta'} = 0$  should be excluded.

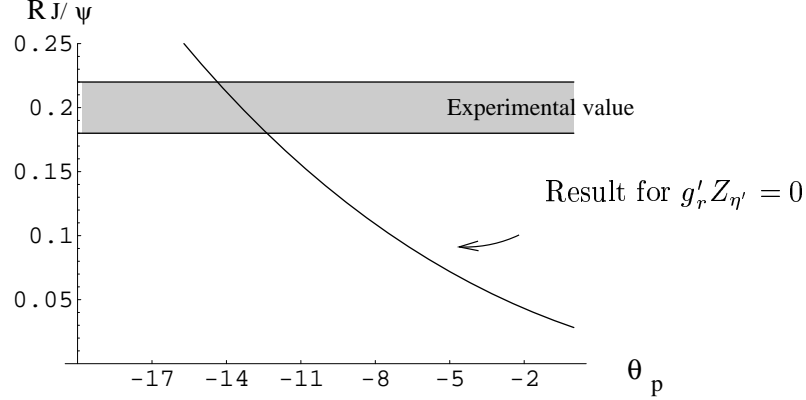


Figure 4: The determination of  $\theta_p$ , putting  $g'_r Z_{\eta'} = 0$  (no gluonic admixture in  $\eta'$ ). The result conflicts with the observation in Section 4.2 when  $g'_r \neq 0$ .

Now let us examine the case of  $g'_r Z_{\eta'} \neq 0$  in Eq. (27). Since we do not know the value of  $g'_r$  which denotes the coupling of two gluons to gluonium we fix the  $\theta_p$  angle at  $-17^\circ$ ,  $-14^\circ$  and  $-11^\circ$  and examine each case. We set the value of  $Z_{\eta'}$  at the maximum which is allowed in Section 4.2. Substituting the left hand side of Eq. (27) for the experimental data, we determine the  $r$  value for each  $\theta_p$  angle. The result is shown in Figure 5. We observe that  $r$  reaches a maximum of 0.3 when  $\theta_p$  is  $-17^\circ$  with 6% of the glue content. That is, the amplitude of the process  $J/\psi \rightarrow \eta' \gamma$  has a maximum contribution of 20% from gluonium in  $\eta'$ .

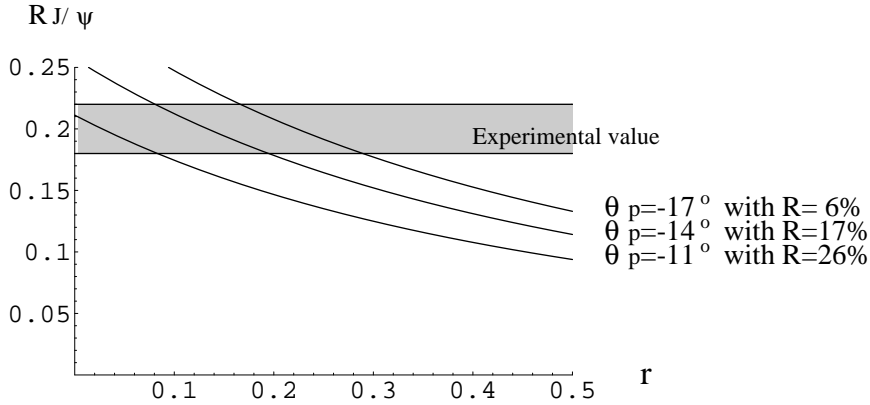


Figure 5: The amplitude of the process  $J/\psi \rightarrow \eta' \gamma$  has a maximum 20% of contribution from gluonium in  $\eta'$  when we choose  $\theta_p = -17^\circ$  with  $R = 6\%$ .

## 6 Conclusion

We have examined the gluonic component of  $\eta'$  and the contributions to the process  $gg \rightarrow \eta'$ . By analysing the latest experimental data on the radiative light meson decays, we have observed that the maximum 26 % of the gluonic component in  $\eta'$  is possible at  $\theta_p = -11^\circ$ . Our constraint on the pseudoscalar mixing angle is  $-17^\circ < \theta_p < -11^\circ$ . Further investigation would be done once the data from DAΦNE will come out. We have also studied the contributions of gluonium to the radiative  $J/\psi$  decays. Combining the obtained result from the analysis on the radiative light meson decays, we found that the  $J/\psi$  decays also demand gluonium in  $\eta'$ . In a case when we choose  $\theta_p = -17^\circ$  with 6% of gluonium in  $\eta'$ , we have observed that the 20% of the amplitude of  $J/\psi \rightarrow \eta'\gamma$  comes from gluonium.

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